15 Weak Quantifier Variance and Mathematical Objects

15.1 A Different Approach to the Unity of Mathematics

So much for Nominalism. Now let's turn to the philosophy of mathematics I ultimately want to advocate.

In Chapter 10, we considered a "unity of mathematics" argument that philosophers who accept the Potentialist set theory advocated in this book should also be Nominalists about other mathematical objects, cashing out all of pure mathematics as an investigation of pure logical possibility facts (i.e., claims stateable without any quantifiers occurring outside of unsubscripted $\Box$ and $\Diamond$ claims). I suggested (very briefly) that a neo-Carnapian approach which combined realism about mathematical objects outside of set theory with Potentialist set theory could also honor these mathematical uniformity intuitions.

In this chapter, I will develop such a proposal, which I call the Weak Quantifier Variance Explanation of Mathematicians' Freedom (QVEMF). It appeals to a Weak Quantifier Variance thesis in philosophy of language, which takes inspiration from some familiar and popular neo-Carnapian ideas, but (as we will see) does not require endorsing controversial Carnapian rejections of metaphysics.

As noted in Chapter 10, adopting such a view lets us honor Benacerraf's idea that we should treat apparently grammatically and inferentially similar talk of numbers, cities and electrons similarly, and avoid classic Quinean and Reference explaining challenges, since we acknowledge that mathematical objects literally exist. However, in Section 15.3 I will note that a kind of Grounding Indispensability challenge arises and discuss some ways of avoiding or answering this challenge.

In Chapter 16 I will consider some more general concerns about developing and defending Weak Quantifier Variance.

15.2 Weak Quantifier Variance and Mathematicians' Freedom

15.2.1 Motivations

To motivate and introduce the Weak Quantifier Variance thesis which supports the approach to mathematics I want advocate, consider our knowledge of holes and shadows.

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In ordinary contexts we appear to quantify over objects like holes in a road or in a piece of Swiss cheese. For example, we may say that there are three potholes in the road between one town and another, or that one piece of cheese has more holes in it than another. And if one accepts the existence of these holes, it is appealing to think of them as distinct from things like the air that occupies them or surrounding portions of the "hole host" (e.g., the cheese or the pavement).[1]

Is there an access problem about our knowledge of holes? One might try to get such an access worry going, by arguing as follows. Our ability to visually determine how many holes there are in a road, depends on our (implicit or explicit) accuracy concerning how hole facts supervene on facts about the distribution of solid matter in space.[2] For example, we must be disposed to make correct judgments about how steeply indented a road must be to count as containing a hole. But what can explain the match between our beliefs on this topic and the corresponding objective reality about when there is a hole in the road? It doesn't seem like sensory experience or scientific practice strongly motivates thinking that any particular place to draw the line is intrinsically physically/metaphysically special (even allowing for some vagueness). Thus, people's apparent ability to draw the line correctly (re: how steeply indented a substance has to be to contain a hole) could seem to create an access problem.[3] Consider the match between *facts* of the form below and human *beliefs* about these facts:

When a road is missing a cylinder of material of depth 3 cm and width 15 cm, there is a hole in that road.

When a road is missing a cylinder of material of depth 0.01 cm and width 0.1 cm, there is *not* a hole in that road.

We process visual information in way that draws the line somewhere (maybe with some vagueness), but what explains the match between where we do draw the line and the correct place to draw the line?

However, it's appealing to say that there isn't really any such access problem for holes, because one can give the following metasemantic explanation for human accuracy about minimum hole indentation facts and the like. If we had been inclined to say something (logically coherent but) different about when an indented object counts as "containing a hole" (e.g., that substances surfaces had to contain an indentation of greater/lesser steepness in order to contain a hole) then the meaning of the words "hole" and "there is" would have been different, so that our utterances would have still expressed truths. That is, we would have been speaking as slightly different language in which a slightly different collection of sentences of the form "Whenever a solid road is indented according to a geometrical formula ϕ, there is a hole in it"

[1] See Lewis (1990). [2] Though see Berry (2019a) for a puzzle about this notion of solidity.

[3] Note that the issue with this "access problem for holes" is not supposed to be about vagueness, but about our ability to be accurate (or even close to accurate) about how hole facts supervene on indentation facts.

express true propositions.[4] Accordingly, there's no mystery or spooky Leibnizian predetermined harmony in our possession of true beliefs about things like about how steeply indented holes must be.[5]

Note that this explanation seems to involve (a form of) quantifier meaning change, in that it requires that our adopting different hole attribution practices would have caused a shift in the meaning of some of our logical vocabulary like the existential quantifier (not just a shift in the meaning of the world "hole"). For example, note that changing between more and less generous standards for hole existence could require the truth-value of the Fregean sentence which says "There are n things" using only first-order logical expressions and equality.[6]

15.2.2 Introducing Weak Quantifier Variance

Thinking about cases like those in Section 15.2.1 motivates the following Weak Quantifier Variance Thesis:

(Weak) Quantifier Variance Thesis:
- There are a range of different meanings "there is" could have taken on, which all obey the syntactic rules for existential quantification.[7]
- These senses need not all be mere quantifier restrictions of some fundamental maximally natural quantifier sense (if there is one).[8]

I call this claim the *Weak* Quantifier Variance thesis because it doesn't include a further "parity" claim (that none of these variant quantifier senses is somehow metaphysically

[4] Arguably our current language allows for contextual variation in how strict the standards for hole existence are and hence (for the reasons to be discussed below) corresponding variation in the meaning of "there is." So one might think of there being a shared core meaning to "there is" (perhaps associated with the introduction and elimination rules) which combines with contextual factors to determine truth conditions for sentences involving "there is" at each metaphysically possible worlds. For present purposes I'll simply talk about shifts in quantifier meaning, but I don't mean to prejudge this issue.

[5] Or at least there's no mystery if we bracket access worries about knowledge of logical coherence.

[6] For example, the sentence that says there are two things $(\exists x)(\exists y)[\neg x = y \wedge (\forall z)(z = x \vee z = y)]$.

Also note that the quantifier meaning change involved explanation above does not suggest that when we start talking in terms of holes and shadows (or switch from stricter to laxer standards for hole existence) we bring these objects into being. The existence of holes and shadows is not caused by, or grounded in, the existence of language users who talk in terms of holes and shadows, and it will be true to say "there were holes before there were people, and before I started talking in terms of them." Instead we are merely changing our language so that some sentences, e.g., "there is something [namely, a hole] in the region of the cheese plate which is not made of matter" go from expressing a false proposition in our old language to expressing a different, true, proposition in our current language (see Einheuser (2006) for a vigorous development of this point).

[7] By this I mean that, for each such quantifier sense there is some possible language such that all applications of the standard syntactic introduction and elimination rules for the existential quantifier within that language are truth preserving. However, that does not mean that one can form a single language containing both quantifier senses and then apply the introduction and elimination rules to prove the equivalence of these senses. See Warren (2014), among others, on this point.

[8] That is, these variant quantifier senses need not be interpretable only as ranging over some subset of the objects which exist in the fundamental quantifier sense, in the way that we might say the "all" in a typical utterance of "all the beers are in the fridge" restricts a more generous quantifier sense to only range only over objects in the speakers' house.

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special) which is generally included in definitions of Quantifier Variance.[9] So, for example, it would be compatible with Weak Quantifier Variance to say that there's a maximally natural quantifier sense corresponding to what objects exist fundamentally.

And, indeed, some friends of traditional metaphysics have found their own reasons for accepting the Weak Quantifier Variance thesis (and thereby putting themselves in a position to give the Quantifier Variance Explanation of Mathematicians' Freedom defended in this chapter). For example, Sider (2009) uses Weak Quantifier Variance to capture the intuition that ordinary speakers' non-philosophical utterances like "There's a hole in the road" can express uncontroversially true statements, even if it's an open question whether holes exist in the sense more relevant to the (traditional fundamental) metaphysics seminar. Sider says there's a unique, maximally natural, sense of the quantifier which ontologists aim to study/employ.[10] But he allows that there are also other (perhaps less metaphysically joint-carving) senses, which the quantifier can take on in ordinary contexts, on which utterances of "There is a hole in the road" clearly can express a true proposition.

Note that saying some kinds of objects (e.g., cities, numbers) might not exist in the sense relevant to the Sider's fundamental ontology room doesn't amount to saying that these objects "don't really exist." It is entirely compatible with truthful assertion that these objects literally exist in the course of daily life (and while studying ethics or non-fundamental metaphysics about money and gender, or writing philosophy of mathematics books like this one) – much as acknowledging that rabbits don't exist on the (relatively) more natural and joint-carving quantifier sense employed by fundamental physics is compatible with saying rabbits literally exist in most ordinary contexts, including biology seminars. When outside the fundamental physics/ontology room, our position on such objects seems much more naturally expressed by saying that rabbits/holes/cities/numbers *might not be fundamental* than that they *don't really exist.*[11]

[9] See, for example, Eklund (2009), Hirsch (2010) and Chalmers' characterization of Quantifier Variance as (roughly) the idea that, "there are many candidate meanings for the existential quantifier (or for quantifiers that behave like the existential quantifier in different communities), with none of them being objectively preferred to the other" (Chalmers 2009).

[10] See the argument that (even from Sider's point of view) we don't *actually speak* a language with Sider's maximally joint-carving quantifier sense in most philosophical contexts (including discussions of metaphysics and ontology).

[11] Also note that (as discussed in Berry (2015)) accepting the Weak Quantifier Variance Thesis does not require one to accept that normal English employs verbally different expressions corresponding to at least two different quantifier senses (a metaphysically natural and demanding one and a laxer one) at the same time, so that it might be true to say things bad-sounding things like "composite objects exist but they do not really exist" in certain contexts. With regard to any particular context, we can fully agree with David Lewis that, "The several idioms of what we call 'existential' quantification are entirely synonymous and interchangeable. It does not matter whether you say 'some things are donkeys' or 'there are donkeys' or 'donkeys exist.' whether true or whether false all three statements stand or fall together" (Lewis 1990).

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15.2.3 Explaining Mathematicians' Freedom

Now let's turn to the special case of mathematics. Contemporary mathematical practice seems to allow mathematicians significant freedom to introduce new kinds of mathematical objects, such as complex numbers, sets and the objects and arrows of category theory. For example, Cole (2013) writes:

> Reflecting on my experiences as a research mathematician, three things stand out. First, the frequency and intellectual ease with which I endorsed existential pure mathematical statements and referred to mathematical entities. Second, the freedom I felt I had to introduce a new mathematical theory whose variables ranged over any mathematical entities I wished, provided it served a legitimate mathematical purpose. And third, the authority I felt I had to engage in both types of activities. Most mathematicians will recognize these features of their everyday mathematical lives.

Philosophers of mathematics face a challenge about how to account for this, and they have developed a number of styles of response.[12]

If we accept the Weak Quantifier Variance Thesis, we can explain mathematicians' freedom to introduce new kinds of apparently coherent objects along the following lines:

> **Quantifier Variance Explanation of Mathematicians' Freedom:** When mathematicians (or scientists or sociologists) introduce axioms characterizing new types of objects, this choice can not only give meaning to newly coined predicate symbols and names but can change/expand the meaning of expressions like "there is," in such a way as to ensure the truth of the relevant hypotheses. Thus, for example, mathematicians' acceptance of existence assertions about complex numbers might change the meaning of our quantifiers so as to make the sentence, "There is a number which is the square root of –1" go from expressing a falsehood to expressing a truth. Similarly, sociologists' acceptance of ontologically inflationary conditionals like, "Whenever there are people who . . . there is a country which . . ." can change the meaning of their quantifiers so as to ensure that these conditionals will express truths.[13]

Hitherto, I take it, versions of QVEMF have largely been developed by philosophers who combine acceptance of the Weak Quantifier Variance thesis with some strong anti-metaphysical claim (such as the parity claim referenced in Section 15.2.2) or project.[14] However, I'm suggesting that more metaphysically realist philosophers could also adopt QVEMF (backed by the Weak Quantifier Variance Thesis) and should consider doing so.

Adopting the Weak Quantifier Variance Thesis and accepting the existence of mathematical objects together with the QVEMF explanation for mathematicians' freedom lets us honor Benacerraf's goal of treating apparently grammatically and inferentially similar talk of numbers and cities similarly (acknowledging that both apparent kinds of objects exist). It allows us to say that a single notion of existence is relevant to claims like "Evelyn is prim" and "Eleven is prime" in any given context

[12] I will say something about how we might generalize this to the case of applied mathematical knowledge (like principles concerning sets with ur-elements) in the next chapter.
[13] See Berry (2015, 2019b).
[14] Here I have in mind Rayo (2015) and Thomasson (2015) as well as Hirsch (2010).

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(though, of course, future choices may further change which notion of existence one's language employs). Proponents of this view need not say that mathematicians' statements are literally false,[15] or say that mathematical statements have a different logical form from claims which ordinary speakers treat similarly (e.g., apparent existence claims about holes and countries), in cases where the specific reasons (like the Burali-Forti worries in Chapter 2) for not doing so.

Admittedly many questions can be raised about Quantifier Variance and the Quantifier Variance explanation of mathematicians' freedom, which I can't discuss at any length here. For example, what would happen if mathematicians simultaneously adopted a pair of internally consistent, but incompatible, conceptions of pure mathematical structures? What would happen if mathematicians adopted a conception of some mathematical structure which imposed undue constraints on the total size of the universe (e.g., a logically coherent collection of axioms describing a purported mathematical structure which imply that the total universe contains at most 100 things)?

In a nutshell, I think we can answer the first challenge by saying that mathematicians' actual (and claimed) freedom only allows a given mathematical community/context to employ any logically coherent *total collection* of conceptions of pure mathematical structures.[16] So, a proponent of the Quantifier Variance explanation of mathematicians' freedom can say that if mathematicians simultaneously employ a pair of incompatible conceptions of mathematical structures (in some context), (a) this would be an accident and (b) at most one of these conceptions of mathematical structures would express a truth.

We can answer the second challenge by noting that axioms characterizing pure mathematical objects always employ quantifiers that are implicitly restricted to some collection of pure mathematical structures (see the discussion of implicit quantifier restriction in pure mathematics in Berry (2018b)), so these conceptions cannot impose any restrictions on the total size of the universe.

However, a fuller answer to these challenges would fit these claims into a general metasemantic story which also yields attractive verdicts about our practice of talking in terms of objects like holes, cities, contracts, etc. I'll sketch such a story in the next chapter.

15.3 Grounding Indispensability Worries for QVEMF

Accepting that mathematical objects (outside set theory) literally exist lets my Quantifier Variantist dodge the classic Quinean and (finitary) reference Indispensability worries for Nominalists. However, the neo-Carnapian realism about mathematical objects I advocate is deeply similar to the forms of Nominalism

[15] Recall Lewis (1991) saying, "I am moved to laughter at the thought of how presumptuous it would be to reject mathematics for philosophical reasons. How would you like the job of telling the mathematicians that they must change their ways, and abjure countless errors, now that philosophy has discovered that there are no classes?"

[16] Thanks to Tom Donaldson for helpful discussion on this point.

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discussed in Chapter 10 (in various ways noted in that chapter). And something can feel troubling about the idea that mere language change can dissolve such a difficult problem.

The Grounding Indispensability argument against the QVEMF below develops this intuition. Note that (as per the Siderian picture in Section 11.4.2) this argument takes some notion of grounding (not necessarily the same one) to apply to: facts, objects and relations. Thus, we can talk about both whether facts involving mathematical objects are grounding fundamental and whether relations like "*x* stands in mass ratio *r* to *y*" are grounding fundamental.

Grounding Worry for Quantifier Variantists: All facts can be grounded in terms of facts involving only fundamental objects.[17] And (one might think!) accepting the Quantifier Variance explanation of mathematicians' freedom requires saying that all logically coherent characterizations of mathematical structures are "on par." Thus, proponents of the QVEMF must either say that all possible logically coherent mathematical structures are metaphysically fundamental (contra the core intuitions used to motivate Potentialist set theory in Chapter 2 or that no mathematical objects are metaphysically fundamental (e.g., all mathematical objects' existence is grounded in modal facts about logical possibility in some way that implies). So, it should be possible to ground all facts involving mathematical objects in facts that don't involve mathematical objects. But, what can ground facts about physical magnitudes if not a relation to numbers? Furthermore, the requirements for nominalistically grounding applied mathematical facts are very similar to those for nominalistically paraphrasing applied mathematics facts. So, Quantifier Variantist realists about mathematical objects face a grounding indispensability problem which is just as bad as the reference indispensability problem.

Thus, one might conclude that the arguments about physical magnitude statements pose a serious problem for the Quantifier Variance realist as well as for the Nominalist.

I will argue that there are a number of attractive strategies for responding to this worry. First, of course, you might argue that the nominalistic Reference and Grounding challenge are both solvable. For example, various philosophers have advocated accepting platonic physical mass, charge and other abstracta. If such platonic physical magnitude abstracta existed, they could be used to answer both finitary Reference and Grounding worries about physical magnitude facts.[18]

Second, you could argue that the Grounding challenge is answerable while the Reference challenge is not. Recall that we had independent reason for thinking formal constraints on grounding are quite different from those specified for adequate paraphrase in Section 11.3.2. The finiteness and learnability constraints on nominalistic paraphrase don't apply when providing grounding. So certain arguments that we can't "adequately" nominalistically regiment physical magnitude statements for the purposes of finite reference explaining challenge don't work when applied to grounding.

For positive examples of answers to the Grounding challenge which aren't answers to the Reference challenge, see Hellman's story about how physical magnitude facts could be grounded in facts involving infinitely many different length/mass/whatever atomic properties. One might also suggest that the "language of metaphysical fundamentalia" is sufficiently different from the languages humans speak, that fundamental facts about the extent to which something is F need not be grounded in facts about

[17] c.f. Sider (2011)'s purity thesis.

[18] One could deploy the strategy.

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whether or not some binary property or relation holds between objects at all. Maybe what's metaphysically fundamental is analog, where language is binary, so to speak.

Third, you could reject the demand for grounding all together. And fourth, you could reject the parity reasoning in Section 15.2.2 above, the idea that someone who gives a quantifier variance explanation of mathematical objects is committed to saying that no mathematical objects are among the metaphysical fundamentalia (none "exist" on Sider's maximally natural quantifier sense). In the rest of the chapter, I will argue that the latter two styles of answers are more appealing than they might at first seem.

15.3.1 Rejecting Grounding

As Quantifier Variance has traditionally been developed as part of a larger neo-Carnapian program which rejects traditional metaphysical questions as meaningless, I suspect that rejecting demands for grounding all together would be most popular response to the Grounding challenge among my fellow neo-Carnapians.

Admittedly, this rejection may seem to come with a cost. For the notion of grounding provides one way of fleshing out an enduringly appealing idea: that an apparently complex universe and variegated language can be explained in terms of a few simple notions. Advocates of the Sideran framework reviewed in Section 11.4.2 will say there's a single small collection of maximally fundamental concepts and kinds of objects, such that facts about them ground everything.

However, neo-Carnapians can and have honored the same idea in a different way, by saying that (in some sense) everything can be reconceptualized in terms of a conceptually parsimonious basis language, but there are a range of different equally good basis languages (perhaps making different choices of mathematical ontology) at issue. We appeal to something like Augustin Rayo's symmetric "nothing but" relation (Rayo 2015) or talk of conceptual re-carving. And we then say that reality is "simple" in the sense that all facts expressible in our language (and maybe some specified range of other languages) bear this nothing-but relation to facts in some simple "basis language."

If we adopt this strategy (i.e., cash out metaphysical parsimony intuitions in terms of something like Rayo's symmetric "just is" relation rather than a grounding relation), we won't say there's a unique correct choice of basis language (the point of the metaphor of basis vectors is that there are a number of different choices which are equally good for representing a given vector space). Rather we can say that a range of choices of basis language are equally capable of bringing out the unity and elegance underlying the diversity and variety we see in the world.

On this strategy, the neo-Carnapian could grant that Nominalists' problems answering the Reference Indispensability Challenge discussed in Chapter 14 reveal that we need to think about physical magnitudes in terms of a relationship to *some* abstract objects (be they numbers on their own, numbers identified with certain sets, or the abstract mass objects, when choosing a parsimonious basis language adequate for stating a simple Theory of Everything). But they could say that all these ways of thinking in terms of different abstract objects are equally good choices of a simple basis language for drawing all the meaningful distinctions we want to draw and showing how a simple shared reality

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lurks under the apparently complexity suggested by natural languages (by paraphrasing natural language statements into some simpler language).

In this way, the neo-Carnapian can claim to achieve whatever the traditional Platonist thinks they've achieved (in terms of the Siderean unifying ambition) by *grounding* everything in a few things by showing that everything *stands in a just is/ conceptual re-carving* etc. relation to a simple basis language.

They will take any traditional Platonist story about what the supposedly metaphysically fundamental objects and concepts are (backed up with the kind of systematic paraphrases of sentences in an apparently richer language with sentences in an apparently narrower one) and say: that's one acceptable basis language. Whatever range of dappled and variegated facts the Platonist thinks are grounded in these few simple facts can indeed be adequately conceptualized in terms of this more limited language/facts/ideology.

However (a neo-Carnapian of this stripe can say) a different basis language which replaced the pure mathematical structures which this paraphrase strategy appeals to with different ones that can do the same applied mathematical work (e.g., replacing appeal to a free-standing copy of the natural numbers or reals with a copy of the numbers inside the hierarchy of sets) would be equally illuminating and "metaphysically insightful" (to whatever extent the neo-Carnapian will grant the meaning of the expression). Any sufficiently expressive pure mathematical language can be combined with some small collection non-pure-mathematical vocabulary to form an adequate basis language.

Note that this idea that different choices of pure mathematical structures (with sufficient expressive power) are somehow philosophically/metaphysically on par[19] fits naturally with a point from the literature on Quinean empiricist answers to access worries about mathematical objects. This is the idea that Quinean indispensability considerations don't seem to justify belief in *any particular* mathematical structure, as different mathematical structures seem capable of doing the same work in regimenting our physical theories[20] and physicists don't seem to care much which ones are invoked.

15.3.2 Agnostic Platonism

Now I want to draw attention to a different, less familiar, style of approach to the Grounding challenge, which I'll call Agnostic Platonism. Friends of the Quantifier Variance explanation of mathematicians' freedom who *don't* share traditional Carnapian opposition to metaphysics can take a different line in responding to the Grounding worry (that would not be available to Nominalists).

Suppose we grant that the history of debate over Quine's indispensability argument suggests some mathematical objects are among the metaphysical fundamentalia. Proponents of the QVEMF can still resist the Grounding worry by rejecting the idea that QVEMF implies all coherent conceptions of mathematical objects must be metaphysically (as opposed to merely mathematically) on par, and thence the argument that no mathematical objects can be grounding fundamental.

[19] See Sections 16.3 and 16.4 for some caveats and a way of thinking this through more carefully.
[20] See, for example, Clarke-Doane (2012).

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In slogan form, someone who accepts agnostic Platonism would say: maybe some mathematical structures are metaphysically special, but mathematicians don't care which ones those are, and they don't need to care in order to reliably form true mathematical beliefs and satisfy the epistemic aims of the project of pure mathematics!

Perhaps indispensability arguments suggest that *some* mathematical objects (capable of doing certain applied mathematical work) exist fundamentally. But, as noted in Section 15.3.1, these considerations don't seem to justify belief in any particular mathematical structure – as different mathematical structures seem capable of doing the same work in regimenting/grounding our physical theories.

Allowing (in response to indispensability worries) that some mathematical structures may be metaphysically fundamental might seem to raise access worries (over and above the access worries about access to facts about logical coherence which the QVEMF theorist already faces[21]). For although these worries can suggest the fundamentalia plausibly include *some* mathematical objects, we don't know (and perhaps can never know) *which*.

But the agnostic Platonist avoids this access problem by saying that getting mathematics right doesn't require guessing which mathematical structures are among the fundamentalia. Note that this idea (that reliably speaking the truth in mathematical ordinary language doesn't require knowing the right answer to corresponding metaphysical questions about fundamental ontology) mirrors what it is natural to say about our knowledge of holes, in the following sense. It may turn out to be the case that some particular hole-like notion (maybe the topological notion of holes) will be used in physics. But construction workers can draw the line where they want with regard to hole boundaries and reliably speak the truth without having to take any such stance regarding fundamental metaphysics.

One might object that a similar access worry arises with regard to metaphysicians' knowledge of which mathematical structures are grounding fundamental. However, we can answer this access worry by noting that there's no access to account for. Metaphysicians don't even *appear* to know very much about which mathematical structures are metaphysically fundamental!

At this point a reader sympathetic to conventional Actualist set-theoretic foundationalism might object: how can I endorse the arbitrariness-based criticism of Actualist set theory developed in Chapter 1, while advocating Agnostic Platonism about mathematical fundamentalia without hypocrisy? For isn't dividing up the pure mathematical objects into those with fundamental existence vs. those without just as arbitrary as saying that the hierarchy of sets just happens to stop at a certain point? And isn't being committed to arbitrariness in which mathematical objects are fundamental just as bad as being committed to arbitrariness in size of the total mathematical universe?

Even if this charge of hypocrisy were correct, I think the Quantifier Variantist view advocated in this chapter would still be an improvement on conventional set-theoretic foundationalism. For the arbitrary joint posited by the agnostic Platonist doesn't constrain acceptable mathematical practice, whereas that posited by classic set-theoretic

[21] See Berry (2018b) for an argument that these access worries about logical coherence are solvable.

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foundationalism does. The agnostic Platonist need not admit any limits on which logically coherent pure mathematical structures mathematicians could choose to talk in terms of. For they don't think mathematicians can only introduce or study structures which are grounding fundamental. In contrast, the conventional set-theoretic Actualist foundationalist holds that any conception of a pure mathematical structure mathematicians could legitimately adopt must have an intended model within the Actualist hierarchy of sets (thus constraining the space of legitimate structures mathematicians could adopt).

However, I will now sketch a more aggressive defense against this charge of hypocrisy. If the other assumptions needed for my Weak Quantifier Variance realist to face access worries hold (i.e., we need to provide grounding, and mathematical objects are needed for that task) then it seems that everyone, not just the Agnostic Platonist, must admit that certain mathematical structures are special in that they play a role in grounding non-mathematical facts about the world (e.g., maybe length reflects a fundamental facet of reality and length facts require grounding in the real numbers).

So the Agnostic Platonist still has the advantage that it only requires us to posit that one special joint in the space of coherent conceptions of mathematical structures (specifying which particular mathematical structures play a role in grounding and/or constituting particular applied mathematical facts, e.g., facts about events and probability, or lengths) where the classic set-theoretic foundationalist is committed to positing two joints in reality (this joint, plus the joint determining where the hierarchy of sets happens to stop). That is, both philosophers will be committed to some kind of fact like "the pure mathematical objects which play roles in grounding physical facts are exactly the real numbers and three layers of sets over them." But the set-theoretic foundationalist will also be committed to a fact like "the hierarchy of sets just happens to stop at X point" (where that point is usually taken to be large enough to accommodate all sets used in our physical theories).

Moreover, it seems more plausible that facts about the fundamental laws of physics might provide an, as yet undiscovered, principled division between those mathematical objects which play a role in grounding applied mathematical facts and those which don't, than it does that some choice of a height for the hierarchy of sets will turn out to be principled.[22]

[22] Indeed, one might argue as follows. Applied mathematics hasn't seemed to motivate a unique choice of which mathematical structures exist, because (from a traditional Platonist point of view) the total collection of mathematical objects must do two jobs. It must make sense of applied mathematics *and* everything we could study in pure mathematics. Given this goal, it has seemed natural to consider both, e.g., both a free-standing real number structure and a copy of the real numbers within various larger structures, like the hierarchy of sets (containing objects for pure mathematical study), as candidates for mathematical reference within our best physical theories. And there's no uniquely natural choice of a collection of mathematical objects which does both jobs.

However, the agnostic Platonist does not expect fundamental mathematical objects to do both these jobs. (As noted above) they can take the truth of existence claims about pure mathematical objects to be grounded in something like facts about logical possibility. Thus, it seems more plausible that whatever aspects of our best physical theories make appeal to fundamental mathematical objects indispensable (if such there are) should suggest a unique choice of which mathematical structures to take to be grounding fundamental.

Thus, to summarize, I think the (admittedly prima facie strange) idea of saying that, although mathematicians can introduce any pure mathematical structure they like, some pure mathematical structures are metaphysically special and instantiated by objects which are grounding fundamental is more appealing than it first seems.

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